GRADE 12
DIPLOMA EXAMINATION

Mathematics 30

January 1988

Alberta
EDUCATION

GRADE 12 DIPLOMA EXAMINATION
MATHEMATICS 30

DESCRIPTION

Time: 2½ hours

Total possible marks: 65

This is a **CLOSED-BOOK** examination consisting of two parts:

PART A: 52 multiple-choice questions each with a value of 1 mark.

PART B: Three written-response questions for a total of 13 marks.

A mathematics data booklet is provided for your reference. Approved calculators may be used.

GENERAL INSTRUCTIONS

Fill in the information on the answer sheet as directed by the examiner.

For multiple-choice questions, read each carefully and decide which of the choices BEST completes the statement or answers the question. Locate that question number on the answer sheet and fill in the space that corresponds to your choice. USE AN HB PENCIL ONLY.

Example				Answer Sheet			

This examination is for the subject area of

                                                      A   B   C   D

A. Chemistry   ①   ②   ③   ●
B. Biology
C. Physics
D. Mathematics

If you wish to change an answer, please erase your first mark completely.

For written-response questions, read each carefully, show all your calculations, and write your answer in the space provided in the examination booklet.

> NOTE: The perforated pages at the back of this booklet may be torn out and used for your rough work. NO MARKS will be given for work done on the tear-out sheets.

DO NOT FOLD EITHER THE ANSWER SHEET OR THE EXAMINATION BOOKLET

The presiding examiner will collect the answer sheet and examination booklet for transmission to Alberta Education.

JANUARY 1988

PART A

1. In $\triangle ABC$, $\angle A = 60°$, $a = 10$ cm, and $b = 5$ cm. The measure of $\angle C$ to the nearest degree is

   A. 94°

   B. 90°

   C. 63°

   D. 26°

2. In the triangle shown at the right, AC is 50 cm. The length of BD to the nearest centimetre is

   A. 30 cm
   B. 23 cm
   C. 21 cm
   D. 19 cm
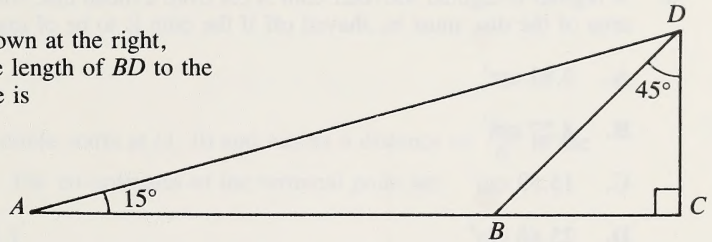
3. Town B is 12 km due east of town A. Town C lies to the north of a line through A and B, and is 10 km from A and 8 km from B. The measure of $\angle A$ to the nearest degree is

   A. 37°

   B. 41°

   C. 60°

   D. 83°

4. In the figure shown at the right, the length of DB to the nearest centimetre is

   A. 53 cm
   B. 56 cm
   C. 59 cm
   D. 65 cm
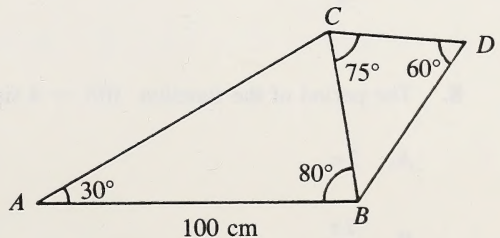
5. A surveyor sets his transit so that it is 50.0 m from the base of one telephone pole and 78.0 m from the base of another. The transit turns through an angle of 20° between the sightings to each pole. The distance between the telephone poles to the nearest tenth of a metre is

    A.   18.4 m
    B.   26.8 m
    C.   28.5 m
    D.   35.4 m

6. A regular octagonal souvenir coin is cut from a metal disc with a radius of 3 cm. What area of the disc must be shaved off if the coin is to be of maximum size?

    A.   2.82 cm^2

    B.   8.27 cm^2

    C.   15.53 cm^2

    D.   25.46 cm^2

7. The terminal arm of an angle in standard position makes 3.5 revolutions clockwise. The angle generated has a radian measure of

    A. -7π

    B. $-\dfrac{7\pi}{2}$

    C. 7π

    D. $\dfrac{7\pi}{2}$

8. The period of the function $f(\theta) = 4 \sin(5\theta + \pi)$ is

    A. 2π

    B. $\dfrac{2\pi}{5}$

    C. $-\dfrac{\pi}{5}$

    D. $-\pi$

9. The graph to the right is a sketch of the function

A. $y = 10 \sin\left(\frac{1}{2}\theta\right)$

B. $y = \sin(5\theta)$

C. $y = 5 \sin\left(\frac{1}{3}\theta\right)$

D. $y = 5 \sin(3\theta)$
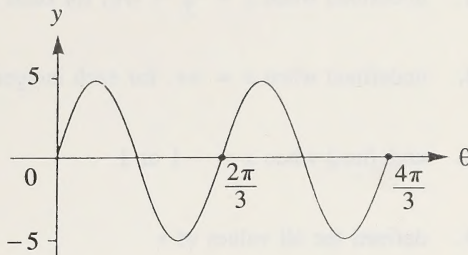
10. Point P on a unit circle starts at $(1, 0)$ and moves a distance of $\frac{5\pi}{6}$ in the positive direction. The co-ordinates of the terminal point are

A. $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

B. $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

C. $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

D. $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

11. If the terminal point of $\angle A$ on a unit circle in standard position is at $(0.6, -0.8)$, then

A. $\tan A = -\frac{4}{3}$

B. $\tan A = -\frac{3}{4}$

C. $\cot A = -\frac{4}{5}$

D. $\cot A = \frac{3}{5}$

12. The function $f(x) = \tan x$, where x is in radians, is

   A.   undefined when $x = \dfrac{\pi}{2} + n\pi$, for each integer n

   B.   undefined when $x = n\pi$, for each integer n

   C.   undefined when $x = -1$ or 1

   D.   defined for all values of x

13. The solutions for the equation $4\cos\theta = \sqrt{12}$, $0° \le \theta < 360°$, are

   A.   30°, 60°

   B.   30°, 330°

   C.   60°, 120°

   D.   60°, 300°

14. An equivalent expression for $\dfrac{\csc\theta\,\tan\theta}{\sec\theta\,\cot\theta}$ is

   A.   1

   B. $\tan^3\theta$

   C. $\tan\theta$

   D. $\cot\theta$

15. If $\sin\theta = -\dfrac{3}{5}$, $\pi < \theta < \dfrac{3\pi}{2}$, the value of the expression

$\tan^2\theta - \sec^2\theta - \cos^2\theta - \sin^2\theta$ is

   A. $\dfrac{9}{8}$

   B.   1

   C.   0

   D. -2

16. In order for $\dfrac{2 - \sin^2 \theta}{\cos \theta} = \cos \theta + x$ to be an identity, x must be

    A. $\sec \theta$

    B. $\sin \theta$

    C. $\csc \theta$

    D. $\sin^2 \theta$

17. The equation of a hyperbola with one vertex at $(0, 8)$ and one asymptote of $y = 4x$ is

    A. $\dfrac{y^2}{64} - \dfrac{x^2}{4} = 1$

    B. $\dfrac{x^2}{64} - \dfrac{y^2}{4} = 1$

    C. $\dfrac{x^2}{4} - \dfrac{y^2}{64} = 1$

    D. $\dfrac{y^2}{4} - \dfrac{x^2}{64} = 1$

18. A hyperbola centred at the origin has foci at $(0, \pm 6)$ and a transverse axis 6 units long. The equation of this hyperbola is

    A. $36x^2 - 9y^2 = 324$

    B. $36y^2 - 9x^2 = 324$

    C. $27x^2 - 9y^2 = 243$

    D. $27y^2 - 9x^2 = 243$

19. The equation of a circle with centre $(3, -2)$ and tangent to the line given by $y - 3 = 0$ is

A. $(x - 3)^2 + (y + 2)^2 = 5$

B. $(x - 3)^2 + (y + 2)^2 = 25$

C. $(x + 3)^2 + (y - 2)^2 = 16$

D. $(x + 3)^2 + (y - 2)^2 = 1$

20. The centre and radius of the circle $x^2 + y^2 + 6x + 2y = -6$ are

A. $(-3, -1);\ 2$
B. $(-3, -1);\ 4$
C. $(3, 1);\ 2$
D. $(3, 1);\ 4$

21. If the equation of an ellipse is $25x^2 + 9y^2 = 225$, then the length of the minor axis is

A. 3
B. 5
C. 6
D. 10

22. If the points $(2, 1)$ and $(3, -4)$ are on a parabola whose equation is $y^2 = bx + c$, the values of b and c respectively are

A. 1 and -3
B. 3 and -5
C. 15 and -29
D. 17 and -33

Use the following information to answer questions 23 to 25.

Equations of Conic Sections

I $16x^2 + 9y^2 = 144$

II $y^2 = 16(x + 4)$

III $x^2 + y^2 = 16$

IV $9x^2 - 16y^2 = 144$

---

23. An ellipse is represented by equation

     A.   I
     B.   II
     C.   III
     D.   IV

24. The conic sections which have the same y-intercepts are

     A.   I and II
     B.   I and III
     C.   II and IV
     D.   III and IV

25. The number of points of intersection for the graphs of curves II and III is

     A.   0
     B.   1
     C.   3
     D.   4

---

26. The expression $\displaystyle\sum_{k=1}^{10} (3 + 2^k)$ is equal to

    A. 1027
    B. 1054
    C. 2038
    D. 2076

27. The convergent sequence is

    A. $10, 8, 6, 4, \ldots, (12 - 2n), \ldots$

    B. $5, 1, \dfrac{1}{5}, \dfrac{1}{25}, \ldots, 5^{(2-n)}, \ldots$

    C. $4, -4, 4, -4, \ldots, 4(-1)^{n+1}, \ldots$

    D. $3, 9, 27, 81, \ldots, 3^n, \ldots$

28. A pendulum bob travels 100 cm on its first swing, and the length of each subsequent swing decreases by $\dfrac{1}{10}$ of the preceding one. The total distance the pendulum bob will travel before coming to rest is

    A. 450 cm
    B. 500 cm
    C. 900 cm
    D. 1000 cm

29. The $\displaystyle\lim_{x \to \infty} \left(\dfrac{100x - 5x^2}{2x^2 + 10x} \right)$ is

    A. $-\dfrac{1}{2}$

    B. $-\dfrac{5}{2}$

    C. 0

    D. 10

30. The common difference of the sequence defined by $t_n = 3n - 5, n \epsilon N$ is

    A. -5
    B. -2
    C. 3
    D. 5

31. In a geometric sequence, the 5th term is 45 and the 9th term is $\dfrac{5}{9}$. The positive common ratio of the sequence is

    A. 9

    B. 3

    C. $\dfrac{1}{3}$

    D. $\dfrac{1}{9}$

32. In the arithmetic sequence 7, 13, 19, . . . , the largest four-digit term is the

    A. 1664th term
    B. 1665th term
    C. 1666th term
    D. 1667th term

33. Every year a student invests $2000 into an account that pays interest at the rate of 7% per annum compounded annually. The accumulated amount (principal plus interest) immediately following the third deposit is

    A. $6420.00
    B. $6429.80
    C. $6879.89
    D. $7350.26

Chemistry	Physics
$\sigma = 17\%$	$\sigma = 6\%$
$\mu = 63\%$	$\mu = 63\%$

34. The results of two tests written by a group of students are shown by the two normal curves above. Sally scored 72% in Chemistry and 67% in Physics. Which of the following statements is true?

   A. Sally's class standing is higher in Physics than in Chemistry.
   B. Sally's class standing is higher in Chemistry than in Physics.
   C. Sally did equally well in Chemistry and Physics.
   D. There is insufficient information to compare Sally's class standings on the two tests.

35. If a set of data approximates a standard normal distribution, then the

   A. mean, mode, and median are equal
   B. mean, mode, and median are different
   C. standard deviation approaches zero
   D. standard deviation approaches infinity

36. A test mark of 78 has a corresponding z-score of 1.6. If the standard deviation is 4, the class mean is

   A. 71.6
   B. 76.4
   C. 79.6
   D. 84.4

37. For the set of data 22, 16, 23, 18, 19, 21, 14, if the standard deviation is $\sqrt{\dfrac{x}{7}}$, then the value of x is

    A.   133
    B.   64
    C.   19
    D.   3

38. A student who wrote five Grade 12 diploma examinations had scores of 62, 58, 65, 72, and 58. The median of the scores is

    A.   58
    B.   62
    C.   63
    D.   65

39. The masses of students were measured to the nearest kilogram and the results are presented in the histogram to the right. If a student is selected at random, the probability (to the nearest 0.01) that his mass will be 65 kg or more is

NUMBER OF STUDENTS

MASSES (kg)

    A.   0.61
    B.   0.54
    C.   0.46
    D.   0.39
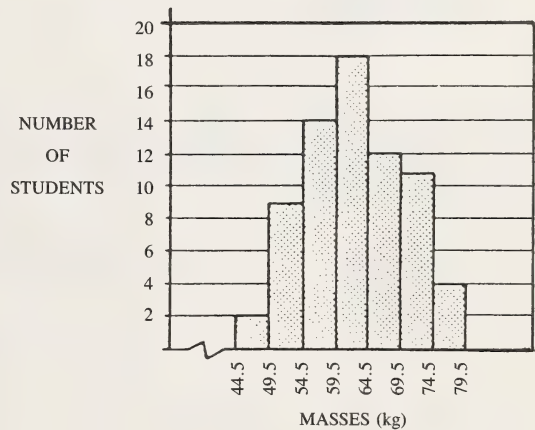
40. Three coins are tossed simultaneously. The probability that the coins will land with two tails and one head showing is

    A. $\dfrac{2}{3}$

    B. $\dfrac{5}{8}$

    C. $\dfrac{3}{8}$

    D. $\dfrac{1}{4}$

41. The lengths of hockey games are normally distributed with a mean of 2.5 h and a standard deviation of 0.25 h. If there are 80 games in a season, how many games can be expected to last longer than 2.2 h?

    A. 32
    B. 48
    C. 71
    D. 79

42. The length of time required to complete a school's scholarship examination is normally distributed with a mean of 60 min and a standard deviation of 17.5 min. If only 85% of the students completed the examination, then the time allowed for the examination was

    A. 78 min
    B. 80 min
    C. 85 min
    D. 87 min

43. The mean life of a motor is 46 months with a standard deviation of 4.3 months. If a normal distribution is assumed, the probability that a motor will last from 46 months to 48 months is

    A. 0.16
    B. 0.18
    C. 0.19
    D. 0.20

44. If $\log_2(8) = x,$ then x is

    A. $\dfrac{1}{3}$

    B. $\dfrac{1}{4}$

    C. 3

    D. 4

45. If $(4)^{\log_{10}(x)} = \dfrac{1}{16}$, then x is equal to

   A. -2

   B. $\dfrac{1}{100}$

   C. 2

   D. 100

46. For the equation $\log_5(25) - \log_{10}(x) = 5$, the value of x is

   A. 10^7

   B. 10^3

   C. 10^{-3}

   D. 10^{-7}

47. In the equation $2^{(x-5)^2} = 4^{x^2 - 5x}$, the value of x is

   A. 5 or -5
   B. 5 or 0
   C. 5
   D. -5

48. If $\log_2(10m + n) = 5$ and $\log_6(10m + 3n) = 2$, then the value of m is

   A. 2
   B. 3
   C. 5
   D. 6

49. The estimated number of bacteria present in a certain culture after t hours is given by $N(t) = c \times 2^{\frac{t}{d}}$, where c is the initial number of bacteria and d is the doubling period. A particular culture of 500 has a doubling period of 2 h. To the nearest tenth of an hour, the time required for the population to reach 10 000 is

    A.   2.2 h
    B.   8.4 h
    C.   8.6 h
    D.   8.8 h

50. A function, $f(x)$, whose graph has x-intercepts $-1, 2, -\frac{1}{2}, \frac{2}{3}$ is

    A. $6x^4 + 17x^3 + 7x^2 - 8x - 4$

    B. $6x^4 - 19x^3 + 13x^2 + 4x - 4$

    C. $6x^4 + 7x^3 - 13x^2 - 4x + 4$

    D. $6x^4 - 7x^3 - 13x^2 + 4x + 4$

51. If $x + 2$ is a factor of $x^3 - 5x^2 + 2x - k$, then k is equal to

    A. -32
    B. -8
    C. -2
    D. 32

52. A complete factorization of $x^3 + 3x^2 - x - 3$ is

    A. $(x - 1)(x + 1)(x + 3)$
    B. $(x - 1)(x + 1)(x - 3)$
    C. $(x - 1)(x - 1)(x + 3)$
    D. $(x + 1)(x + 1)(x - 3)$

YOU HAVE NOW COMPLETED THE MULTIPLE-CHOICE SECTION OF THE EXAMINATION. PLEASE PROCEED TO THE NEXT PAGE AND ANSWER THE WRITTEN-RESPONSE QUESTIONS IN PART B.

PART B

TOTAL MARKS: 13

START PART B IMMEDIATELY

1. If $P(x) = x^3 + mx^2 - 5x + k$, $P(-1) = 8$, and $P(2) = -4$, determine the values of m and k.

$m =$ _____ $k =$ _____

2. The first term of an arithmetic series is 87 and the last term is 12.

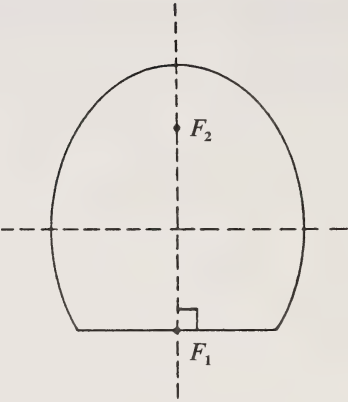**a.** How many terms are there in this series if its sum is 792?

$n\ =$ _____

b. Find the common difference.

$d\ =$ _____

3. The shape of the opening to a tunnel is the part of the ellipse as sketched in the diagram. The base of the tunnel passes through the lower focus of the ellipse. The distance between the foci is 8 m and the maximum width of the tunnel is 6 m. What is the width of the tunnel at its base?

The width is _____

YOU HAVE NOW COMPLETED THE EXAMINATION. IF YOU HAVE TIME, YOU MAY WISH TO GO BACK AND CHECK YOUR ANSWERS.

FOLD AND TEAR ALONG PERFORATION

FOLD AND TEAR ALONG PERFORATION

NAME: ☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐

(LAST NAME)

☐☐☐☐☐☐☐☐☐☐☐☐☐☐

(FIRST NAME)

DATE OF BIRTH: ☐☐ Y ☐☐ M ☐☐ D   SEX: ☐

PERMANENT MAILING ADDRESS: _____

(Apt./Street/Ave./P.O. Box)

_____

(Village/Town/City)

_____
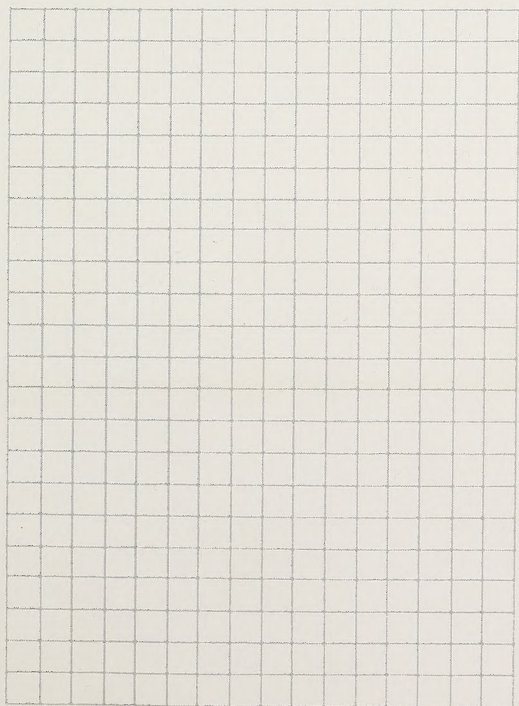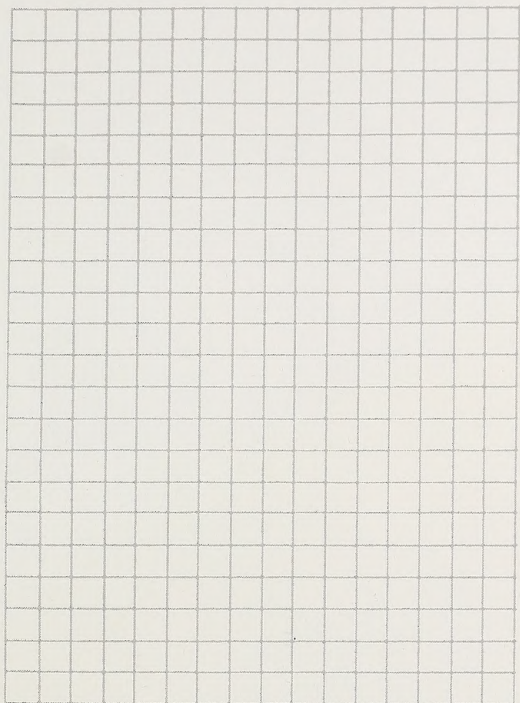
(Postal Code)

SCHOOL CODE: ☐☐☐   SCHOOL: _____

SIGNATURE: _____

MATHEMATICS 30